

Hybrid imploding scalar and ads spacetime.

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Abstract

A solution to the massless scalar cosmological constant field equations is presented. The solution has imploding scalar and parts of anti-deSitter or deSitter spacetime as limiting cases. Some of the solutions properties are discussed however not much can be said because of the contrasting properties of imploding scalar and deSitter spacetimes.

Imploding scalar solutions have found application in models of gravitational collapse [1, 2, 3]. Since the discovery of imploding spacetime [4, 5] much so far unsuccessful effort has gone into attempting to add additional parameters such as mass, rotation and electric charge. Here a solution is given which adds a cosmological constant to the original imploding solution: there is no explicit interaction between the scalar field and the cosmological constant in the sense that the cosmological constant does not appear in the scalar field.

The field equations are taken to be

$$R_{ab} = \Lambda g_{ab} + 2\psi_a\psi_b, \quad (1)$$

where Λ is the cosmological constant and ψ a scalar field. In double null coordinates the solution is

$$ds^2 = \Omega^2 [-dudv + r_+r_-d\Sigma_2^2], \quad d\Sigma_2^2 \equiv d\theta^2 + \sin(\theta)^2d\phi^2, \quad (2)$$

$$2r_{\pm} \equiv (1 \pm 2\sigma)u - v, \quad \Omega_{\pm} = (1 \pm \frac{\Lambda uv}{12})^{-1}, \quad \psi = \frac{1}{2} \ln \left(\frac{r_-}{r_+} \right),$$

where σ is the scalar charge and $r = r_+$, $\Omega = \Omega_-$. The solution has a conformal Killing potential

$$K = cuv\Omega, \quad K \cdot K = -2cuv\Omega^2, \quad K_{a;b} = -2c\frac{\Omega}{\Omega_+}g_{ab}, \quad (3)$$

where c is a constant; note that K can be null, spacelike or timelike. For $\Lambda \neq 0$, $K_a = 12c\Omega_a/\Lambda$ and Ω_a can be taken to be the conformal Killing vector. The first derivative of the scalar field ψ obeys

$$\psi \cdot \psi = \frac{\sigma^2 uv}{\Omega^2 r_+^2 r_-^2}, \quad K \cdot \psi = 0, \quad (4)$$

with which using the field equations (1) invariants constructed from the Ricci tensor can be constructed; also the Weyl scalars are given by $3\Psi_2 = \psi \cdot \psi$. A null tetrad is

$$n_a = -\delta_a^u, \quad l_a = \frac{1}{2}\Omega^2 \delta_a^v, \quad m_a = \Omega\sqrt{r_+ r_-} \left(\delta_a^\theta + \sin(\theta)\delta_a^\phi \right). \quad (5)$$

The null surface projection, second fundamental form, normal fundamental form, surface gravity and surface stress are defined by

$$\begin{aligned} q_{ab} &\equiv g_{ab} - 2l_{(a}n_{b)}, \quad \chi_a^{(l)} \equiv l_{d;c}q_a^c q_b^c, \quad \eta_a^{(l)} \equiv l_{d;c}q_a^c n^d, \quad \omega^{(l)} \equiv -l_d n^c n_{;c}^d, \\ \tau_{ab}^{(l)} &\equiv -\chi^{(l)c}{}_c n_a n_b - 2\eta_{(a;b)}^{(l)} - \omega^{(l)} q_{ab}, \end{aligned} \quad (6)$$

respectively. In the present case just the first term contributes to the surface stress giving

$$\tau_{ab}^{(l)} = -\frac{\Omega}{2r_+ r_-} \left(v - u + \frac{\Lambda}{12} v^2 (u - (1 - 4\sigma^2)v) \right) \delta_{ab}^{uu}. \quad (7)$$

At $v = 0$ the scalar field ψ vanishes and the metric is the same as flat spacetime, the easiest way to see this is to note that $g_{\theta\theta} = r^2 \exp(2\psi)$ then when substituting $\psi = 0$ this gives flat spacetime. Now

$$\tau_{ab}^{(l)}|_{v=0} = \frac{\Omega u}{2r_+ r_-} \delta_{ab}^{uu}, \quad (8)$$

and as this is independent of both σ and Λ the junction is well-defined.

Topics not looked at include the following. The use of conformal factors to find further solutions [7]. The properties of the Lanczos potential [9], which presumably are inherited from the $\Lambda = 0$ case because of the vanishing of the Weyl tensor in deSitter spacetime. Extension to higher dimensions, this has been done for imploding scalar solutions [8]. Embedding in higher dimensions, although this can be done in general for spherically symmetric spacetimes a simple embedding for imploding scalar spacetime is not known. The properties of geodesics: deSitter spacetime is one of the few spacetimes in which all geodesics can be easily expressed [6], imploding scalar spacetime

is exactly the opposite not only is there no simple expression for the global geodesics but furthermore local expressions for the world function and van Vleck determinant converge slowly. The global properties of the solution (3), although the global properties of the limiting cases are known when the familiar coordinate transformations are applied one gets off-diagonal metric components and so forth. Group properties: although the group properties of deSitter and anti-deSitter spacetime are well known nothing is known about the group properties of imploding scalar spacetime or of (3).

In conclusion a solution which has both imploding scalar and parts of anti-deSitter or deSitter spacetime as limiting cases was presented and the null junction conditions shown to be well behaved.

References

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